8. THE THEORY OF RECEIVING ANTENNAS

The receiving aerial is destined for reception of electromagnetic waves and transformation of their energy to the energy of directed waves or currents of the high frequency. The operation principle of the reception aerial differs from the operation principle of the transmitting aerial. To show this, let us place the aerial in the field of the electromagnetic wave. It is obvious, that under action of a tangential component of the intensity vector of the electric field in every aerial element EMF will be induced. In contrast to the transmitting aerial where the applied EMF is concentrated between input terminals, in the reception aerial electromotive forces are allocated all over the surface.

Due to the occurrence of the dispersed EMF in the reception aerial the current starts flowing. This results in the inducing of the secondary field, excited by the reception aerial. The intensity of the secondary field may be found from the boundary condition: the sum of tangential intensity components of primary and secondary fields on the antenna surface should be equal to zero. Such secondary field, a reradiating field, is typical for the aerial, which operates in the reception mode.

Thus, the aerial operation in the reception mode essentially differs from the operation in the transmitting mode. These features of reception aerials result in that their theoretical research is more complex in comparison with the transmitting aerials. In this connection for the definition of the basic properties of the reception aerials the reciprocity principle is applied. According to it characteristics of the aerial, that functions in the reception mode, is determined from parameters of the same aerial operating in the transmitting mode.

8.1 The dipole in the field of the flat electromagnetic wave

The mechanism of the EMF occurrence on the dipole terminals under action of an electromagnetic field may be considered by means of the strict theory, which is stated in subsection 4.3. If to assume, that the current distribution in the reception aerial is the same as in the transmitting aerial, the analysis of dipole operation in the reception mode becomes simpler.

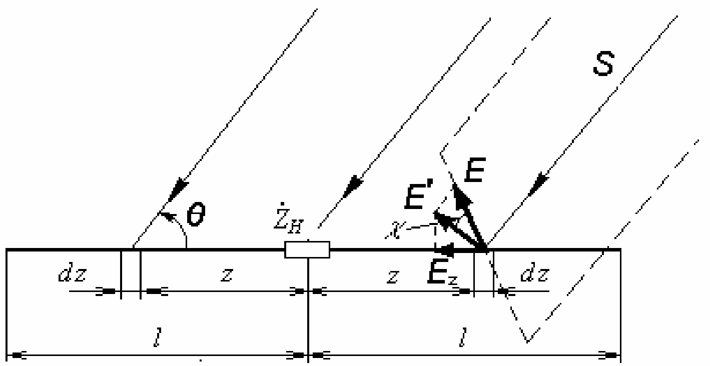

Fig. 8.1

Let us find EMF on terminals of the dipole, placed in a field of a flat wave. Generally, the polarization plane of an incident wave does not coincide with a plane, which is combined with an axis of the dipole and direction of the wave arrival. In Fig. 8.1 the dipole with length $2l$, loaded on impedance Z_H is presented. In the plane of figure there is the axis of the radiator and the line of the direction of the wave arrival. The polarization plane of wave *S* is placed under angle χ to the figure plane

$$E' = E\cos\chi .$$

The tangential component of the intensity vector on the surface of the dipole is

$$\dot{E}_Z = E'\sin\theta\, e^{ikz\cos\theta} = E\cos\chi\ \sin\theta e^{ikz\cos\theta} .$$

Here it is accepted, that the phase of tangential component of vector E in the dipole centre $(z = 0)$ is equal to zero.

The electromotive force in elements dz, symmetrically located with respect to the centre of the dipole, is written down as

$$d\dot{\mathcal{E}}_{Z1} = E\cos\chi\sin\theta e^{ikz\cos\theta}dz;$$

$$d\dot{\mathcal{E}}_{Z2} = E\cos\chi\sin\theta e^{-ikz\cos\theta}dz.$$

Sum of EMF in elements dz is

$$d\dot{\mathcal{E}} = d\dot{\mathcal{E}}_{Z1} + d\dot{\mathcal{E}}_{Z2} = E\cos\chi\sin\theta\left(e^{i\,kz\cos\theta} + e^{-i\,kz\cos\theta}\right)dz = \\ = 2E\cos\chi\sin\theta\cos(kz\cos\theta)dz$$

Let us determine the voltage on terminals, which arises owing to the occurrence of EMF in element dz. For this purpose the reciprocity principle is used, its definition is: EMF $\dot{\mathcal{E}}_1$, applied to an input of the linear passive two-port network (Fig. 8.2*(a)*), causes such current $\dot{I}_2$ on an output, as well as current $\dot{I}_1$ on an input at applying the same EMF $\dot{\mathcal{E}}_2$ to an output of the two-port network (Fig. 8.2*(b)*). If EMF $\dot{\mathcal{E}}_1$ is not equal to EMF $\dot{\mathcal{E}}_2$, then the reciprocity principle in the analytical form is written down as the relation

$$\frac{\dot{\mathcal{E}}_1}{\dot{I}_2} = \frac{\dot{\mathcal{E}}_2}{\dot{I}_1}. \tag{8.1}$$

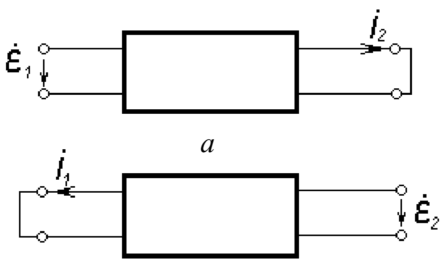

Fig. 8.2

With reference to dipole EMF $d\dot{\mathcal{E}}_A$, applied to its input terminals (Fig. 8.3), causes current $\dot{I}_z$ in element dz, EMF $d\dot{\mathcal{E}}_Z$, excited in element dz, causes current $\dot{I}_A$ on terminals. On the basis of the reciprocity principle (8.1) the following relation can be set:

$$\frac{d\dot{\varepsilon}_A}{\dot{I}_z} = \frac{d\dot{\varepsilon}_Z}{\dot{I}_A}.$$

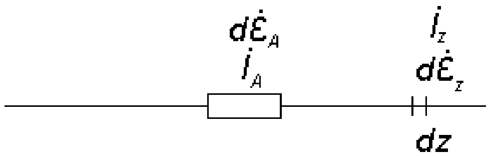

Fig. 8.3

Hence EMF, which arises on dipole terminals with the occurrence of EMF in element dz, is

$$d\dot{\varepsilon}_A = \frac{\dot{I}_z}{\dot{I}_A} d\dot{\varepsilon}_Z.$$

The ratio of current I_z in any section of the dipole to the current on terminals is a function of the current distribution

$$f(z) = \frac{I_z}{I_A} = \frac{\sin k(l - z)}{\sin kl}.$$

The total EMF on dipole terminals is

$$\varepsilon_A = \int_0^l f(z) d\varepsilon_z = \frac{2E\cos\chi\sin\theta}{\sin kl}\int_0^l \sin k(l - z)\cos(kz\cos\theta)dz.$$

After integration it is finally received

$$\varepsilon_A = \frac{E\lambda\cos\chi}{\pi\sin kl}\frac{\cos(kl\cos\theta) - \cos kl}{\sin\theta}. \qquad (8.2)$$

Let us express the obtained *EMF* value through the effective length of the aerial. From formula (4.13) value of the effective length is

$$l_e = \frac{\lambda}{\pi}\frac{1 - \cos kl}{\sin kl}. \qquad (8.3)$$

For the dipole a maximum of non-normalized DC (4.9) is

$$f_{\max}(\theta) = 1 - \cos kl.$$

Therefore normalized dipole DC is

$$F(\theta) = \frac{\cos(kl\cos\theta) - \cos kl}{\sin\theta(1 - \cos kl)}\,.$$

Taking into account the value of normalized DC from formula (8.2) and the value of the effective length (8.3)

$$\mathcal{E}_A = E l_e \cos\chi F(\theta)\,. \qquad (8.4)$$

Thus, EMF on terminals of the dipole, placed in the field of the flat electromagnetic wave, is defined by parameters of the aerial (the effective length and the normalized DC), the intensity of the electric field, as well as by the orientation of the intensity vector of the electric field with respect to the dipole.

8.2. The reciprocity principle in the theory of reception aerials

Application of the reciprocity principle is possible under condition, that characteristics of aerials and medium, in which electromagnetic waves are propagated, do not depend on current, voltage and intensity of the electromagnetic field. Besides, the medium must be isotropic. Such conditions in most cases are completely satisfied.

Let us consider two aerials (Fig. 8.4*(a)*), which are spaced apart significantly. Types and designs of these aerials may be completely different. Let us connect the source of EMF with the internal impedance Z_{L1} to the first aerial. Loading Z_{L2} is supplied to the terminals of the second aerial.

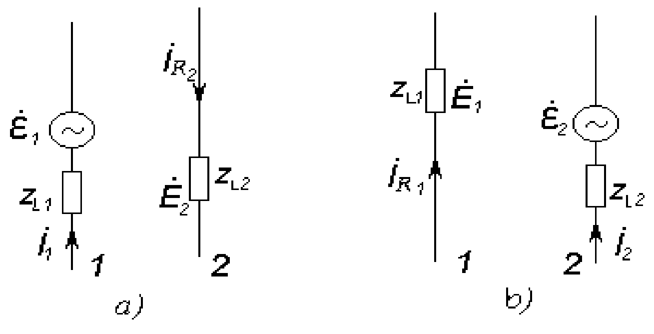

Fig. 8.4

Under the action of $\dot{\mathcal{E}}_1$ in the first aerial current $\dot{I}_1$ will flow, owing to what there will be the radiation field, which intensity electric vector on the surface of the second aerial will have value $\dot{E}_2$. Owing to it current $\dot{I}_{L2}$ will flow in the second aerial.

The electromotive force, which operates between terminals of the first aerial, is

$$\dot{\mathcal{E}}_1 = \dot{I}_1 (Z_{A1} + Z_{L1}), \tag{8.5}$$

where Z_{A1} is the input resistance of the aerial.

The field intensity $\dot{E}_2$ is

$$\dot{E}_2 = A\vec{e}_1(\theta_1, \varphi_1) l_{e1} \dot{I}_1 F_1(\theta_1, \varphi_1) e^{i\psi_1(\theta_1, \varphi_1)}, \tag{8.6}$$

where $A = i\dfrac{30k}{r} e^{-ikr}$; l_{e1} is the effective length of the first aerial, θ_1, φ_1 are coordinates of the second aerial in the spherical system, which origin coincides with the phase centre of the first aerial.

From formula (8.6) values of the current of the first aerial can be found

$$\dot{I}_1 = \frac{\dot{E}_2}{A\vec{e}_1(\theta_1, \varphi_1) l_{e1} F_1(\theta_1, \varphi_1) e^{i\psi_1(\theta_1, \varphi_1)}},$$

and after its substitution in expression (8.5)

$$\dot{\mathcal{E}}_1 = \frac{\dot{E}_2 (Z_{A1} + Z_{L1})}{A\vec{e}_1(\theta_1, \varphi_1) l_{e1} F_1(\theta_1, \varphi_1) e^{i\psi_1(\theta_1, \varphi_1)}}. \tag{8.7}$$

Now let us supply EMF $\dot{\mathcal{E}}_2$ to the terminals of the second aerial; the first aerial will work in a mode of reception (Fig. 8.4, *b*). Similarly,

$$\dot{\mathcal{E}}_2 = \frac{\dot{E}_1 (Z_{A2} + Z_{L2})}{A\vec{e}_2(\theta_2, \varphi_2) l_{e2} F_2(\theta_2, \varphi_2) e^{i\psi_2(\theta_2, \varphi_2)}},$$

(8.8)

where values with index 2 are attributed to the second aerial and have the appropriate components of similar values with index 1.

The principle of reciprocity (8.1) in the theory of receiving aerials is thus formulated: the ratio of EMF $\dot{\mathcal{E}}_1$ on terminals of the first aerial to current $\dot{I}_{R_2}$, induced in the second aerial by the field of the first

aerial, at constant position of aerials is equal to the ratio of EMF $\dot{\mathcal{E}}_2$, on terminals of the second aerial to current $\dot{I}_{R_1}$, which is induced in the first aerial by the field of the second aerial. Using given designations:

$$\frac{\dot{\mathcal{E}}_1}{\dot{I}_{R_2}} = \frac{\dot{\mathcal{E}}_2}{\dot{I}_{R_1}}. \tag{8.9}$$

Substituting values $\dot{\mathcal{E}}_1$ from formula (8.7) and $\dot{\mathcal{E}}_2$ from formula (8.8) in expression (8.9), reducing all values, concerning with the first aerial in the left part, and with the second aerial – in the right part

$$\frac{\dot{I}_{R1}(Z_{A1} + Z_{L1})}{\vec{e}_1(\theta_1, \varphi_1) \dot{E}_1 l_{e1} F_1(\theta_1, \varphi_1) e^{i\psi_1(\theta_1, \varphi_1)}} = \frac{\dot{I}_R (Z_{A2} + Z_{L2})}{\vec{e}_2(\theta_2, \varphi_2) \dot{E}_2 l_{e2} F_2(\theta_2, \varphi_2) e^{i\psi_2(\theta_2, \varphi_2)}}$$

(8.10)

As polarization of vector $\dot{E}_1$ is defined by polarizing properties of the second aerial, and polarization of vector $\dot{E}_2$ - by polarizing properties of the first aerial, and as direction θ_1 and φ_1 in the spherical coordinate system, adhered to the first aerial, determines the angular position of the second aerial, and direction θ_2, φ_2 in the spherical coordinate system, adhered to the second aerial, determines the angular position of the first aerial, then vectors $\dot{E}_1$ and $\dot{E}_2$ can be written down using polarizing multipliers of aerials:

$$\dot{E}_1 = \dot{E}_1 \vec{e}_2(\theta_2, \varphi_2)$$

and

$$\dot{E}_2 = \dot{E}_2 \vec{e}_1(\theta_1, \varphi_1).$$

Substituting values of vectors $\dot{E}_1$ and $\dot{E}_2$ in equation (8.10), in denominators of both parts, we obtain the same scalar product of polarizing vectors

$$e_1(\theta_1, \varphi_1) e_2(\theta_2, \varphi_2).$$

It will be shown later, that the square of the scalar product of polarizing vectors defines the factor of the polarizing coordination.

As it has been specified before, both aerials have been chosen without restrictions on their design and the principle of operation, therefore, the ratio of quantities, which enter the left and the right parts of equation (8.10) do not depend on the type of the aerial and should be equal to some constant C. So, for any aerial it can be written down

$$\frac{\dot{I}_R(Z_A + Z_L)}{\dot{E} l_e F(\theta, \varphi) e^{i\psi(\theta, \varphi)}} = C(\vec{e}_A, \vec{e}_P),$$

where owing to the generalization of the received expression there are designations for polarizing vector e_A of the aerial and for the vector of polarization of the electromagnetic field, in which there is receiving aerial e_P. It is obvious, that for the first aerial $e_A = e_1(\theta_1, \varphi_1)$ and $e_P = e_2(\theta_2, \varphi_2)$, and for the second aerial $e_A = e_2(\theta_2, \varphi_2)$ and $e_P = e_1(\theta_1, \varphi_1)$.

From the generalized expression the current, which flows through terminals of the receiving aerial is

$$\dot{I}_R = C(\vec{e}_A, \vec{e}_R) \frac{\dot{E} l_e F(\theta, \varphi) e^{i\psi(\theta, \varphi)}}{Z_A + Z_L}.$$

The electromotive force on terminals of the receiving aerial is

$$\dot{\varepsilon}_A = C(e_A, e_R)\dot{E} l_e F(\theta,\varphi) e^{i\psi(\theta,\varphi)} . \qquad (8.11)$$

To determine constant *C*, expression (8.11) can be compared with the expression for EMF on terminals of the receiving dipole (8.4).

In Fig. 8.1 angle θ is measured from axis Z of the dipole, therefore the plane of the figure is the meridional plane in the spherical coordinate system. Comparing Fig. 8.1 with Fig. 3.1, it can be noted, that for the dipole, as well as for an electric elementary dipole, when they are plotted along the axis, from which the meridional angle is measured, the unit polarizing vector takes the value

$$e_A = \theta_0 .$$

In an incident wave (see Fig. 8.1) vector E is perpendicular to the direction of the wave propagation and can obtain two cophased components E_θ and E_φ at linear polarization. It is apparent from Fig. 8.1 that $E_\theta = E' = E\cos\chi$. So, the polarization vector of the incident wave is

$$e_P = \theta_0 \cos\chi + \varphi_0 \sin\chi .$$

It is obvious, that for this case

$$(e_A, e_P) = \cos\chi .$$

For the dipole $\psi(\theta,\varphi) = 0$, therefore, comparing the right parts of formulas (8.4) and (8.11)

$$C(e_A, e_P) = \cos\chi .$$

Thus, the constant C is equal to unity.

Taking into account value of the constant C (8.11) and values of the effective length (2.45), finally, the expression for the current of the receiving aerial can be written down

$$\dot{I}_R = \frac{\dot{E}(e_A, e_P)}{Z_A + Z_L}\sqrt{\frac{DR_\Sigma}{30k^2}} F(\theta,\varphi) e^{i\psi(\theta,\varphi)} ,$$

(8.12)

where Z_H is the input impedance of the receiver (the loading impedance of the aerial).

So, the current flowing through terminals of the aerial, which operates in the reception mode, can be defined by parameters of the same aerial, which is in the transfer mode. Therefore parameters of the aerial can be defined in the radiation mode or in the reception mode of electromagnetic waves no matter in what mode the aerial will be used. The directional characteristic, DF of the aerial is identical whether operating on radiation or reception, if the transmitter and the receiver are connected to the same terminals.

The receiving aerial can be considered as a source of EMF, which values are determined from expression (8.11)

$$\dot{\varepsilon}_A = \dot{E}(e_A, e_P) l_e F(\theta,\varphi) e^{i\psi(\theta,\varphi)}, \quad (8.13)$$

and the internal impedance has value Z_A. Formula (8.13) is used to calculate of EMF generally.

At the polarizing matching we obtain $|(e_A, e_P)| = 1$. If to match the aerial polarization and orient the DC maximum in the direction of the wave arrival, then on the aerial terminals EMF will be maximal:

$$\varepsilon_{A\max} = E\, l_e\ . \quad (8.14)$$

From expression (8.14) it is possible to derive a formula for the effective length of the receiving aerial (2.43).

8.3. Power in loading of the receiving aerial

Let us determine the power, which is allocated in loading of the receiving aerial. For this purpose from expression (8.12) the amplitude value of the current, which flows through terminals of the receiving aerial with the loaded complex resistance $Z_L = R_L + iX_L$ can be found:

$$\dot{I}_R = \frac{\dot{E}(e_A, e_P)}{|Z_A + Z_L|} \sqrt{\frac{DR_\Sigma}{30k^2}} F(\theta,\varphi)\ . \quad (8.15)$$

Taking into account formula (2.31) and expression for efficiency (2.18), let us make such replacement in formula (8.15)

$$DR_\Sigma = GR_A, \quad (8.16)$$

where $R_A = R_\Sigma + R_{LOS}$ is the active component of the antenna input resistance.

The load power is

$$P = \frac{1}{2} I_R^2 R_L .$$

Using expressions (8.15) and (8.16) we obtain:

$$P = \frac{E^2 G F^2(\theta,\varphi)}{240k^2} \frac{4R_A R_L}{|Z_A + Z_L|^2} \left[(\vec{e}_A, \vec{e}_P)\right]^2 . \qquad (8.17)$$

For convenience of the further analysis let us enter designations

$$\rho = \frac{4R_A R_L}{|Z_A + Z_L|^2} = \frac{4R_A R_L}{(R_A + R_L)^2 + (X_A + X_L)^2} ; \qquad (8.18)$$

$$a_{pol} = \left[(e_A, e_P)\right]^2 ; \qquad (8.19)$$

$$P_{\max} = \frac{E^2 G}{240k^2} = \frac{E^2 G \lambda^2}{960\pi^2} . \qquad (8.20)$$

Expression (8.17) with the entered designations becomes

$$P = P_{\max} F^2(\theta,\varphi) \rho \, a_{pol} . \qquad (8.21)$$

It follows from formula (8.21), that the load power of the receiving antenna is defined by four factors. First of them (8.20) has the dimension of power, while the others are dimensionless.

The square of the normalized DC is a number, which value depends on the DD orientation in space. At orientation of the DD maximum in the direction of the wave arrival the DC value $F_{\max}(\theta,\varphi) = 1$. Thus, by changing the angular position of the axis or the aperture of the aerial (turn of its DD) it is possible to increase the load power.

Value ρ [see formula (8.18)] characterizes the matching of the input impedance of the aerial with the load impedance. Value ρ reaches its maximum at the full matching

$$Z_A = Z_L^*$$

or

$$R_A = R_L; \quad X_A = - X_L . \qquad (8.22)$$

When condition (8.22) is satisfied, then $\rho = 1$. In other cases $\rho < 1$.

Quantity a_{pol} [see formula (8.19)] represents the factor of the polarizing coordination (2.13). Really, if $F(\theta, \varphi) = 1$ and $\rho = 1$ at $a_{pol} \neq 1$ expression (8.21) will express power $P_{R.Mism.}$, which is absorbed in loading at any polarization of an incident wave, which does not coincide with the polarization of the aerial. If the polarization of the wave correlates with the polarization of the aerial $e_A = e_P$ and the scalar product of unit vectors will be equal to unity, power $P_{R.Match.}$, absorbed at the full polarizing coordination, will be maximal. It is obvious, that if we substitute the specified powers in formula (2.12), we shall derive expression (8.19).

The unit polarizing vector can be considered as a complex vector, which is determined through an intensity vector of the electromagnetic field (1.72) using the relation

$$\dot{\vec{e}}_P = \vec{e}_P e^{i\omega t} = \frac{\dot{E}}{\sqrt{\dot{\vec{E}}\dot{\vec{E}}^*}}.$$

The real part of this complex vector is equal to

$$\operatorname{Re}\left(e_P e^{i\omega t}\right) = \theta_0 \cos\beta \cos\omega t + \varphi_0 \sin\beta \cos(\omega t + \psi_P) \qquad (8.23)$$

and for every moment of time it coincides with the direction of the field intensity vector of radiated waves.

As it follows from definition of the complex polarizing vector

$$\cos\beta = \frac{|E_\theta|}{\sqrt{|E_\theta|^2 + |E_\varphi|^2}}$$

and

$$\sin\beta = \frac{|E_\varphi|}{\sqrt{|E_\theta|^2 + |E_\varphi|^2}}.$$

In the same way we can determine the complex polarizing vector $\dot{e}_A$, using expression for the radiation field of the considered aerial.

Therefore the presented definition of the unit polarizing vector as the complex vector can be written down as

$$\dot{\vec{e}}_A = \vec{\theta}_0 e_{A\theta} + \vec{\varphi}_0 \tilde{e}_{A\varphi} = \vec{\theta}_0 \cos\alpha + \vec{\varphi}_0 \sin\alpha e^{i\psi_A};$$
$$\dot{\vec{e}}_P = \theta_0 e_{P\theta} + \varphi_0 \tilde{e}_{P\varphi} = \theta_0 \cos\beta + \varphi_0 \sin\beta e^{i\psi_P}, \qquad (8.24)$$

where trigonometrical functions of angle α are determined through orthogonal components of vector E^A of the antenna radiation field

$$\cos\alpha = \frac{\left|E_\theta^A\right|}{\sqrt{\left|E_\theta^A\right|^2 + \left|E_\varphi^A\right|^2}}$$

and

$$\sin\alpha = \frac{\left|E_\varphi^A\right|}{\sqrt{\left|E_\theta^A\right|^2 + \left|E_\varphi^A\right|^2}}.$$

Components of the right part of equation (8.23) correspond to two orthogonal linearly-polarized components of the field intensity E_θ and E_φ, therefore the end of vector $e_P e^{i\omega t}$ describes a polarizing ellipse. Connection between parameters of the polarizing vector (8.23) and the polarizing ellipse can be expressed by the equation

$$\mathrm{tg}\,2\gamma = \frac{2\,\mathrm{ctg}\,\beta\cos\psi_P}{\mathrm{ctg}^2\beta - 1}; \qquad (8.25)$$

$$K_e^2 = \frac{1 - \mathrm{ctg}^2\beta\,\mathrm{tg}^2\gamma}{\mathrm{ctg}^2\beta - \mathrm{tg}^2\gamma}. \qquad (8.26)$$

At $\beta = 45^\circ$, as it follows from formula (8.25) $\gamma = 45^\circ$. For these values β and γ the right part of formula (8.26) becomes uncertain. Getting rid of uncertainty

$$K_e = \mathrm{tg}\frac{\psi_P}{2}.$$

The factor of the polarizing coordination (8.19) is equal to the square of the module of the scalar product of the unit polarizing complex vectors (8.24) and its values can vary from zero to unity. If the aerial of the linear polarization is used and the incident wave on the aerial is polarized linearly, then vectors e_A both $e_П$ are real and their scalar product is equal to cosine of a spatial angle between them - $\cos\chi$, where $\cos\chi = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

So,

$$a_{pol} = \cos^2 \chi . \tag{8.27}$$

Generally according to equation (8.24) the polarizing coordination factor comes [19]

$$\begin{aligned} a_{pol} &= \left[\cos\alpha\cos\beta + \sin\alpha\sin\beta\cos(\psi_A + \psi_P)\right]^2 \\ &+ \sin^2\alpha\sin^2\beta\sin^2(\psi_A + \psi_P) = \\ &= \cos^2\alpha\cos^2\beta + \sin^2\alpha\sin^2\beta + \frac{1}{2}\sin 2\alpha\sin 2\beta\cos(\psi_A + \psi_P). \end{aligned} \tag{8.28}$$

It is evident, that the right parts of equations (2.13) and (8.28) express the same quantity through different parameters.

Coming back to expression (8.21), we can see, that in the right part three factors take the maximal values, equal to unity, under such conditions:

- Overlapping of a direction of a maximum of a wave reception with a direction of its arrival $(F_{\max}(\theta,\varphi) = 1)$;

- Matching of the aerial input impedance with the load impedance $(\rho = 1)$;

- At the full polarizing coordination of the aerial with the accepted electromagnetic wave $(a_{pol} = 1)$.

Simultaneous realization of these three conditions results in the allocation of the maximal power in the loading. Its values can be found from expression (8.20). Taking into account formula (2.31), expression (8.20) takes the form

$$P_{\max} = \eta_A \frac{E^2 D \lambda^2}{960\pi^2}.$$

If in the aerial there are no losses, then $\eta_A = 1$. At this value of the maximal power, which is absorbed in the matched loading of the lost-free antenna, is

$$P_{\max} = \frac{E^2 D \lambda^2}{960\pi^2}. \tag{8.29}$$

The same power can be calculated, using the formula for the effective area S_e (2.36) and the formula for the density of a power flux Π in free space (1.77) at $W = 120\pi$:

$$P_{\max} = \Pi S_e = \frac{E^2}{240\pi} S_e. \qquad (8.30)$$

Equating the right parts of expressions (8.29) and (8.30), we can find connection between the directivity factor and the effective area (2.37) (it was previously considered without the proof).

8.4. Noise temperature of the aerial

The operating quality of separate circuits of the acceptance system is frequently estimated by the ratio of a useful signal power to the interference power. Sources of the interference are lightning, space, industrial radiations, radiations of radio stations, thermal noise.

Power of interference can be determined by formula (8.30) at the known power density. But it is more convenient to estimate the obstacle power by the effective noise temperature, which characterizes the power of the thermal noise, generated in the resistor in the matched loading:

$$P_N^{rez} = k\Delta f T, \qquad (8.31)$$

where $k = 1.38 \cdot 10^{-23} \frac{\text{W}}{\text{Hz} \cdot \text{K}}$ is Boltzmann's constant; Δf is the band of frequencies, for which the thermal noise power is defined; T is the absolute temperature.

Let us define in the similar way the noise power, generated in the aerial matched loading. Using expression (8.30) and having determined the effective area from formula (2.37):

$$S_e = \frac{\lambda^2}{4\pi} D.$$

Generally, DF depends on the radiation direction [see formula (2.22)], therefore, the effective area depends on the direction of the electromagnetic wave arrival

$$S_e(\theta,\varphi) = \frac{\lambda^2}{4\pi} D F^2(\theta,\varphi). \qquad (8.32)$$

Let the power density of the noise radiation in borders of unit of a solid angle be equal to $\Pi'_N(\theta,\varphi)$. Then some part of the noise power, allocated in the antenna loading owing to reception of electromagnetic waves in borders of the angle $d\Omega = dS/r^2 = \sin\theta d\theta d\varphi$, is calculated in the following way:

$$dP_N = \Pi'_N(\theta,\varphi)S_e(\theta,\varphi)\sin\theta d\theta d\varphi .$$

The polarization of noise of electromagnetic waves is random, whereas the aerial accepts waves of certain polarization. According to expression (8.21) at $\rho = 1$ it is possible to write down

$$dP_N = \Pi'_N(\theta,\varphi)S_e(\theta,\varphi)a_{pol}\sin\theta d\theta d\varphi .$$

Let us assume, that the aerial of the linear polarization is considered and electromagnetic waves from noise sources are also linearly polarized, but their polarization planes are oriented in space arbitrarily. In this case value a_{pol} is determined from expression (8.27). For noise signals it is possible to consider, that angle χ is random and the probability density of angle χ is uniformly distributed from 0 to π. Therefore, a mean value of power, which is absorbed in the aerial loading at reception of waves and propagates in an element of the solid angle $d\Omega$, can be found from the formula

$$dP_N = \frac{1}{\pi}\int_0^{\pi}\Pi'_N(\theta,\varphi)S_e(\theta,\varphi)\cos^2\chi\sin\theta d\theta d\varphi d\chi = \\ = 0.5\Pi'_N(\theta,\varphi)S_e(\theta,\varphi)\sin\theta d\theta d\varphi .$$

The total noise temperature is

$$P_N = 0.5\int_0^{\pi}\int_0^{2\pi}\Pi'_N(\theta,\varphi)S_e(\theta,\varphi)\sin\theta d\theta d\varphi . \qquad (8.33)$$

The density of the noise power per unit of the solid angle, according to Rayleigh-Jeans law, is equal to

$$\Pi'_N(\theta,\varphi) = \frac{2kT_{env}(\theta,\varphi)\Delta f}{\lambda^2}, \qquad (8.34)$$

where $T_{env}(\theta,\varphi)$ is an effective temperature of the medium in the direction, determined by coordinates θ and φ.

Having substituted expressions (8.32) and (8.34) in equation (8.33) we obtain:

$$P_N = \frac{k\Delta f D}{4\pi} \int_0^{\pi} \int_0^{2\pi} T_{env}(\theta,\varphi) F^2(\theta,\varphi) \sin\theta \, d\theta \, d\varphi \,.$$

(8.35)

At entering concept of the aerial noise temperature T_A the noise power, generated in loading, will be defined similarly to expression (8.31)

$$P_N = k\Delta f T_A \,. \qquad (8.36)$$

Equating the right parts of equations (8.35) and (8.36), the calculation formula for the noise temperature of the aerial can be found:

$$T_A = \frac{D}{4\pi} \int_0^{\pi} \int_0^{2\pi} T_{env}(\theta,\varphi) F^2(\theta,\varphi) \sin\theta \, d\theta \, d\varphi \,. \qquad (8.37)$$

In the case when the effective temperature of the medium in borders of the beamwidth does not depend on angular coordinates, expression (8.37) in view of formula (2.25) takes the form

$$T_A = T_{env} \,.$$

Distribution of the effective temperature $T_{env}(\theta,\varphi)$ depends on the wave band and the position of noise sources. At low frequencies atmospheric and industrial noises prevail, in UHF range - space and thermal obstacles.

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